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Research on Application Characteristics of Fractional Calculus in Sliding Mode Control of Robotic Arm

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Abstract: An effective fractional order sliding mode control method is proposed for the nonlinear and uncertain robotic arm system. In the design process of the controller, fractional calculus is introduced into the sliding mode control by using fractional order reaching law and fractional order sliding mode control law respectively. The stability of the system is proved by Lyapunov theory, and the simulation is carried out by MATLAB. The results show that the proposed control strategy can effectively improve the tracking speed and tracking accuracy of the joints, and the controller has good effectiveness and strong robustness.

1. Introduction

Fractional Introduction calculus is an old mathematical with a 300 years old history. With the rapid development of natural and social sciences, fractional calculus theory has been successfully applied in engineering, physics, system control and other fields in recent decades.

Manipulator is a widely used and typical multi-input and multi-output complex system, which has the characteristics of non-linearity and time-varying uncertainty. The common control methods of this kind of system are PID control[1], adaptive control[2], neural network control[3], robust control[4], sliding mode control[5]. Among them, the sliding mode control system does not need to provide accurate dynamic model in the design process, only need to use the position tracking error of the trajectory to design the sliding mode surface reasonably, and this method has the characteristics of fast response and good robustness. It is found that the introduction of fractional calculus into sliding mode control theory can make the design more flexible[6,7]. In this paper, the application characteristics of fractional calculus in sliding mode control of manipulator are studied from two aspects: fractional order reaching law and fractional order sliding mode control law.

2. Fractional Calculus

Different from the integer order calculus theory, the order of fractional order calculus can be arbitrarily selected, which greatly improves the flexibility of control system design. In the definition, the basic operators of fractional calculus can be represented by ${}_aD_t^\alpha$, where a and t represent the upper limit and lower limit of calculus respectively, while α represents the order, and real or complex numbers can be selected.

The definition of Caputo type is as follows:

$$ {}_aD_t^\alpha = \frac{1}{\Gamma(m-\alpha)}\int_a^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}\,d\tau \qquad (1)$$

where $m-1<\alpha\le m$, m is an integer and $\Gamma(\bullet)$ is a Gamma function. In order to simplify the expression, D^α is used instead of ${}_aD_t^\alpha$ without involving the upper and lower limit of the calculus operator.

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Lemma 1[5]. If $x = 0$ is the equilibrium point of a nonautonomous fractional order system ,

$$D^{\alpha}x(t) = f(x,t) \tag{2}$$

where $f(x,t)$ satisfies Lipschitz condition. Assume that there exist a Lyapunov candidate $V(t,x(t))$ satisfying

$$\begin{aligned} \alpha_1(\|x\|) \le V(t,x(t)) \le \alpha_2(\|x\|) \\ D^{\beta}V(t,x(t)) \le -\alpha_3(\|x\|) \end{aligned} \tag{3}$$

where α_1, α_2 and α_3 are positive constants, $\beta \in (0,1)$. Then the system (2) is asymptotically stable.

3. Fractional Sliding Mode Reaching Law

Different from the traditional sliding mode control theory, this paper chooses a fractional order reaching law. It can be represented as Eq.(4), by changing the order α and coefficient k in the formula, the state of the control system can reach the speed of sliding mode surface.

$$D^{\alpha}s = -k\,\mathrm{sign}(s), 0 < \alpha < 1, k > 0 \tag{4}$$

Proof. The Lyapunov function to be defined in the form

$$V(t) = \frac{1}{2}s^T s \tag{5}$$

According to the definition form of fractional calculus of Caputo type

$$\begin{cases} \dot{s} > 0 \\ \dot{s} < 0 \end{cases} \Rightarrow \begin{cases} D^{\alpha}s > 0 \\ D^{\alpha}s < 0 \end{cases} \tag{6}$$

Taking derivative of both side of Eq.(5) and using Eqs.(4) and (6), we have

$$\dot{V}(t) = s^T s = s^T D^{1-\alpha}(-k\,\mathrm{sign}(s)) \tag{7}$$

Using $\mathrm{sign}(D^{1-\alpha}(-k\,\mathrm{sign}(s))) = -k\,\mathrm{sign}(s)$ [8], one obtains

$$\begin{aligned} \mathrm{sign}(\dot{V}(t)) &= \mathrm{sign}(s^T)\mathrm{sign}(D^{1-\alpha}(-k\,\mathrm{sign}(s))) \\ &= -k\,\mathrm{sign}(s^T)\mathrm{sign}(s) \\ &= -k \end{aligned} \tag{8}$$

then $\dot{V} \le 0 \Rightarrow D^{\alpha}V \le 0$, according to lemma 1, the equilibrium point of the system (4) is asymptotically stable.

4. Fractional Sliding Mode Controller for Manipulator

The dynamic model of *n*-joint manipulator is as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{9}$$

where $q \in R^n$ is the position of the joint, $\dot{q} \in R^n$ and $\ddot{q} \in R^n$ are the velocity and acceleration vectors of the joint, $M(q) \in R^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in R^{n\times n}$ is the centrifugal force and Coriolis force matrix, $G(q) \in R^n$ is the gravity term, and $\tau \in R^n$ is the control moment of the joint. However, there are many uncertainties in applications, which are unified as external disturbances. When expressed in $f(t)$, the system model is as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + f(t) \tag{10}$$

Taking $q_d(t)$ as the ideal position of the joint and $q(t)$ as the actual position of the joint, the position tracking error of each joint is defined as:

$$e(t) = q_d(t) - q(t) \tag{11}$$

The sliding surface is defined as:

$$\begin{aligned} s &= D^{\alpha} e + \lambda e \\ \lambda &= diag(\lambda_1, \lambda_2, \cdots, \lambda_n), \lambda_i > 0 \end{aligned} \tag{12}$$

Taking derivative of Eq.(12), it yields

$$\begin{aligned} \dot{s} &= D^{\alpha-1}\ddot{e} + \lambda \dot{e} \\ &= D^{\alpha-1}(\ddot{q}_d - M^{-1}(\tau + f - C\dot{q} - G)) + \lambda \dot{e} \end{aligned} \tag{13}$$

By selecting the exponential reaching law $\dot{s} = -\varepsilon \mathrm{sign}(s) - ks$ and combining it with Eq.(13), we have

$$\tau = M(\ddot{q}_d + \lambda D^{1-\alpha}\dot{e} + \varepsilon D^{1-\alpha}\mathrm{sign}(s) + D^{1-\alpha}ks) + C\dot{q} + G \tag{14}$$

Proof. The Lyapunov function to be defined in the form

$$V = \frac{1}{2}s^2 \tag{15}$$

Taking derivative of both side of Eq.(15) and using Eq.(12), we have

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s(D^{\alpha-1}\ddot{e} + \lambda \dot{e}) \\ &= s(D^{\alpha-1}(\ddot{q}_d - M^{-1}(\tau + f - C\dot{q} - G)) + \lambda \dot{e}) \end{aligned} \tag{16}$$

Substituting (14) into (16) results in

$$\dot{V} = s(D^{\alpha-1}(-\lambda D^{1-\alpha}\dot{e} - \varepsilon D^{1-\alpha}\mathrm{sign}(s) - D^{1-\alpha}ks) + \lambda \dot{e}) \le -ks^2 - \varepsilon |s| \tag{17}$$

According to Lyapunov stability theory, the system is asymptotically stable. Literature [5] proves that when the parameters are selected properly, the tracking error can converge to zero.

5. Simulation Results

Taking a two-joint manipulator as an example, the FOMCON fractional order modeling and control toolbox[9] is used to complete the numerical simulation. The specific parameters of the mechanism are as follows:

$$M(q) = \begin{bmatrix} v + q_{01} + 2q_{02}\cos(q_2) & q_{01} + q_{02}\cos(q_2) \\ q_{01} + q_{02}\cos(q_2) & q_{01} \end{bmatrix}, \; C(q,\dot{q}) = \begin{bmatrix} -q_{02}\dot{q}_2\sin(q_2) & -q_{02}(\dot{q}_1 + \dot{q}_2)\sin(q_2) \\ q_{02}\dot{q}_1\sin(q_2) & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 15g\cos q_1 + 8.75g\cos(q_1 + q_2) \\ 8.75g\cos(q_1 + q_2) \end{bmatrix}, \; f(t) = 3\sin(2\pi t)$$

where $v = 13.33$, $q_{01} = 8.98$, $q_{02} = 8.75$, $g = 9.8$. The initial state of the system is given as $\begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.3 & -0.5 & 0.5 \end{bmatrix}$, where $\lambda_1 = \lambda_2 = 5$, $\varepsilon = 0.5$, $k = 5$, $0 < \alpha < 1$. Using the reaching law and sliding mode control mentioned above, the following three methods are deduced for simulation experiments.

Method 1. Sliding mode and exponential reaching law

$$\begin{cases} s_1 = \lambda e + \dot{e} \\ \tau_1 = M\left(\lambda \dot{e} + \ddot{q}_d + \varepsilon \mathrm{sign}(s) + ks\right) + C\dot{q} + G - f \end{cases}$$

Method 2. Sliding mode and fractional order reaching law

$$\begin{cases} s_2 = \lambda e + \dot{e} \\ \tau_2 = M\left(\lambda \dot{e} + \ddot{q}_d + kD^{1-\alpha}\mathrm{sign}(s)\right) + C\dot{q} + G - f \end{cases}$$

Method 3. Fractional sliding mode and exponential reaching law

$$\begin{cases} s_3 = D^{\alpha} e + \lambda e \\ \tau_3 = M\left(\ddot{q}_d + D^{1-\alpha}\lambda \dot{e} + \varepsilon \mathrm{sign}(s) + ks\right) + C\dot{q} + G - f \end{cases}$$

When the desired trajectory of a joint is given as a sinusoidal signal, the results are obtained. Figs.1 to 3 show the position tracking and control input of each joint under different methods, and Fig.4 shows the position tracking errors of each joint under three control methods. From the comparison of Figs.1 and 2, it can be seen that choosing the fractional order reaching law can soften the trajectory to a certain extent, accelerate the tracking speed significantly, and reduce the chattering of the system obviously. Figs.1 and 3, it can be concluded that the controller designed with fractional sliding surface has higher control accuracy and better robustness.

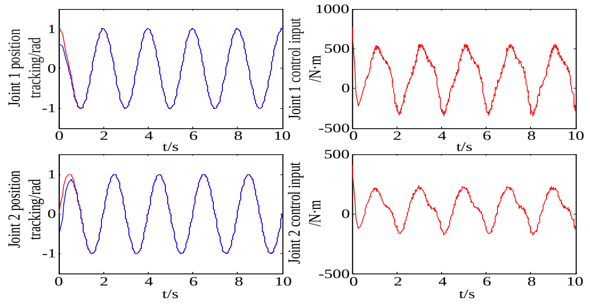

Fig.1 Position tracking and control input curve of method 1

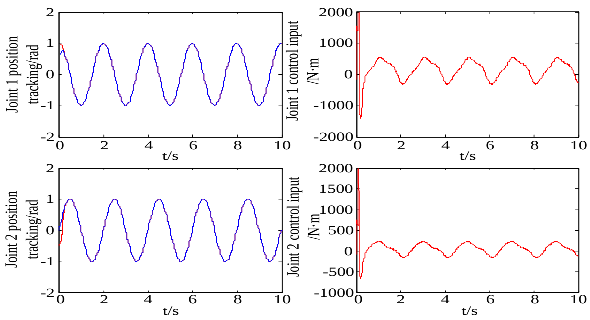

Fig.2 Position tracking and control input curve of method 2

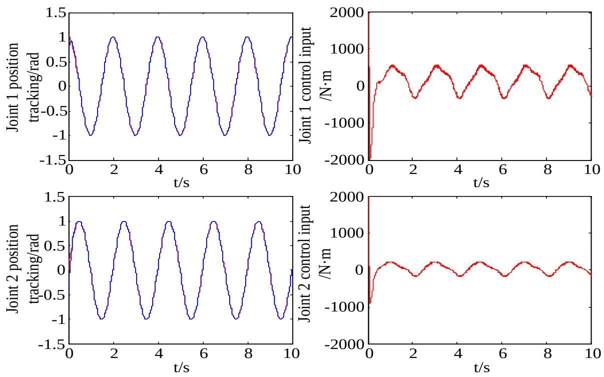

Fig.3 Position tracking and control input curve of method 3

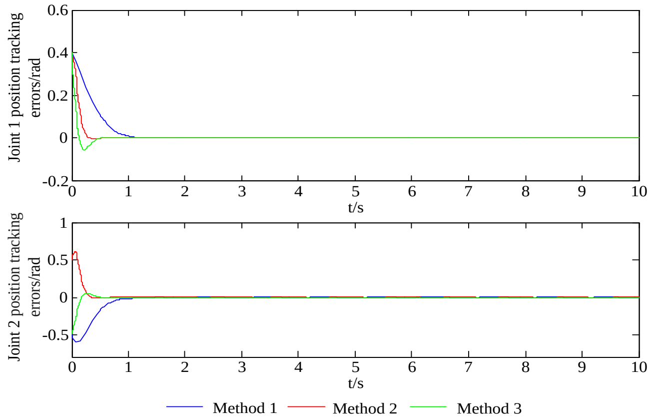

Fig.4 Comparison of position tracking errors of three methods

In order to judge the tracking performance of each control method, angular displacement adjustment time and the root mean square error of position error are selected as the reference values. The so-called angular displacement adjustment time is the time required for the angular displacement from the initial state to the tracking error less than or equal to 0.01 rad. The RMSE is used to judge the following performance. The results of data comparison are shown in Tab.1. It can be concluded from Fig.4 and Tab.1 that the introduction of fractional calculus accelerates the adjustment time of joint angular displacement, while the fractional sliding mode controller has less fluctuation of position tracking error and better tracking effect after joint adjustment.

Tab.1 Data comparison of three control methods

	Method 1	Method 3	Method 3
Adjustment time of joint 1 angular displacement (s)	1.75	0.71	0.52
Adjustment time of joint 2 angular displacement (s)	1.62	0.90	0.62
The RMSE of joint 1 after 2s (rad)	1.55×10^{-4}	4.24×10^{-5}	3.78×10^{-5}
The RMSE of joint 2 after 2s (rad)	8.04×10^{-4}	3.21×10^{-4}	2.11×10^{-4}

6. Conclusion

The simulation results show that the introduction of fractional order reaching law can soften the trajectory, have better smoothness and weaken chattering. And the characteristics of fractional sliding surface are more reflected in the enhancement of system robustness and the improvement of control accuracy.

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